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Azimuthal Repositioning of Payloads in Heliocentric Orbit Using Solar Sails

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I. Introduction

FUTURE solar physics missions will require the ability to reposition multiple spacecraft at different azimuthal positions relative to the Earth, while remaining close to a one year circular orbit. Such azimuthal repositioning will allow stereoscopic views of solar features to be generated and will allow imaging of coronal mass ejections as they transit the sun–Earth line. The NASA STEREO mission, which is scheduled for launch in 2005, will utilize two spacecraft to perform such tasks. Both spacecraft will be launched on a Delta II 7925 and will use multiple lunar gravity assists to maneuver the spacecraft onto leading and trailing heliocentric orbits. The two spacecraft will then drift ahead of and behind the Earth on free-drift trajectories, with increasing Earth–sun–spacecraft angles.

One limitation of such free-drift trajectories is that the Earth–sun–spacecraft angle is uncontrolled. It would be advantageous to stop the spacecraft drift at certain azimuthal positions relative to the Earth to perform various observing campaigns. Different observations require different spacecraft angular separations to optimize the mission science return. In principle, such control is possible by using chemical propulsion to start and stop the azimuthal drift. However, as will be seen, such maneuvers can incur a large accumulated Δv. In this Note, the use of small solar sails is investigated to control the spacecraft azimuthal drift using solar radiation pressure. Because propellant mass is not an issue for solar sails, there are benefits in launch mass reduction over chemical propulsion.

The problem is investigated using analytical methods by linearizing the solar sail two-body equations of motion in the vicinity of the Earth's orbit. When polar coordinates are used, only small departures in orbit radius are required to ensure that the linear solutions provide a good representation of the solar sail trajectory. A simple two-step sail steering strategy is then investigated and the sail attitude is optimized to provide the largest change in azimuthal position relative to the Earth for a fixed maneuver duration. Last, a comparison is made with chemical propulsion, and the benefits of solar sail propulsion for such maneuvers is assessed.

II. Relative Equations of Motion

The heliocentric equations of motion for an ideal, planar solar sail may be written in plane polar coordinates (r, θ) as[1]

$$\ddot{r}(t) - r(t)\dot{\theta}(t)^2 = -[\mu/r(t)^2](1 - \beta\cos^3\alpha) \tag{1a}$$

$$r(t)\ddot{\theta}(t) + 2\dot{r}(t)\dot{\theta}(t) = \beta[\mu/r(t)^2]\cos^2\alpha\sin\alpha \tag{1b}$$

where $r(t)$ is the heliocentric distance of the solar sail from the sun at time t, $\theta(t)$ is the polar angle of the solar sail measured from some reference direction, and μ is the gravitational parameter of the problem. Because both solar radiation pressure and solar gravity have an inverse square variation with heliocentric distance, the sail performance can be parameterized by the sail lightness number β, defined as the ratio of the solar radiation force to the solar gravitational force acting on the solar sail. The total sail mass per unit area σ is related to β using $\beta = 1.53/\sigma$ $(g \cdot \mathrm{m}^{-2})$ (Ref. 1). The sail pitch angle α is defined as the angle between the sun–sail line and the sail normal.

Here, we are interested in trajectories that will result in a large change in polar angle, but only a small change in orbit radius. In orbit to investigate such trajectories analytically, a new set of coordinates will be defined to facilitate the linearization of Eqs. (1), as shown in Fig. 1. The coordinates will be referenced to the instantaneous position of the Earth, given in polar coordinates (r, θ) by $[R, \omega(t - t_0)]$, where R is the orbit radius of the Earth and $\omega = \sqrt{(\mu/R^3)}$ is the orbital angular velocity of the Earth. When defining $\xi(t) = r(t) - R$ and $\phi(t) = \theta(t) - \omega(t - t_0)$, the equations of motion can be linearized to allow analysis using analytical methods.

Equations (1) will now be expanded to first order, so that terms of order $(\xi/R)^2$ and above are neglected. In addition, because only small solar sails will be considered, terms of order $\beta(\xi/R)$ are taken as second order and so are neglected. Neglecting these mixed terms

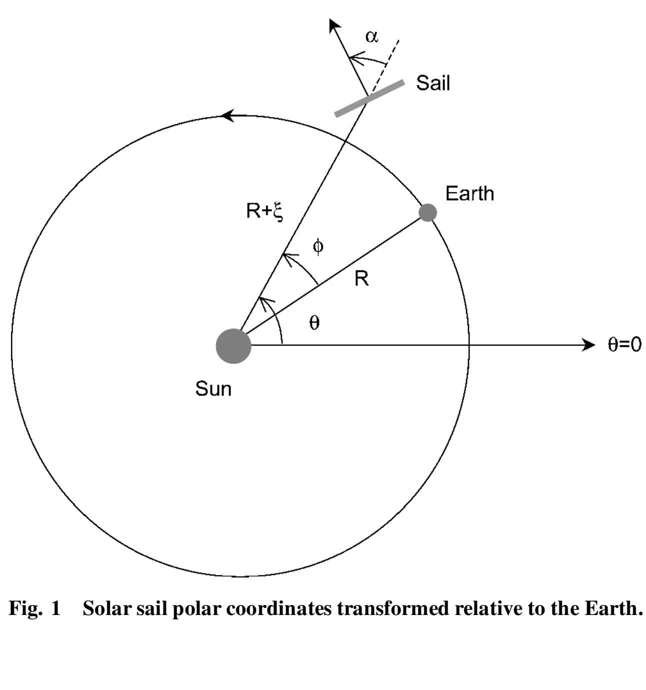

Fig. 1 Solar sail polar coordinates transformed relative to the Earth.

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is not in fact essential to the linearization, but greatly simplifies further analysis. It can be shown that the resulting linearized equations of motions may be now written as

$$\ddot{\xi}(t) - 2\omega R\dot{\phi}(t) - 3\omega^2\xi(t) = a_\xi \tag{2a}$$

$$\ddot{\phi}(t) + (2\omega/R)\dot{\xi}(t) = (1/R)a_\phi \tag{2b}$$

where the forcing terms due to solar radiation pressure are defined as

$$a_\xi = \beta(\mu/R^2)\cos^3\alpha \tag{3a}$$

$$a_\phi = \beta(\mu/R^2)\cos^2\alpha\sin\alpha \tag{3b}$$

It can be seen that Eq. (2) are similar in form to the Clohessy–Wiltshire equations.[2] For a fixed sail pitch angle, Eqs. (2) may now be solved using standard methods to obtain the solar sail coordinates as

$$\begin{aligned}\xi(t) = {} & 4\xi_0 + a_\xi\big/\omega^2 + 2R\dot{\phi}_0/\omega + (2a_\phi/\omega)(t-t_0)\\ & - 3\xi_0\cos[\omega(t-t_0)] - \left(2R\dot{\phi}_0\big/\omega^2\right)\cos[\omega(t-t_0)]\\ & + (\dot{\xi}_0/\omega)\sin[\omega(t-t_0)] - \left(2a_\phi\big/\omega^2\right)\sin[\omega(t-t_0)]\\ & - \left(a_\xi\big/\omega^2\right)\cos[\omega(t-t_0)]\end{aligned} \tag{4a}$$

$$\begin{aligned}\phi(t) = {} & \phi_0 + 4a_\phi\big/\omega^2 R - 2\dot{\xi}_0/\omega R - 3\dot{\phi}_0(t-t_0)\\ & - (6\xi_0\omega/R)(t-t_0) - (2a_\xi/\omega R)(t-t_0) - (3a_\phi/2R)(t-t_0)^2\\ & + (2\dot{\xi}_0/\omega R)\cos[\omega(t-t_0)] + (4\dot{\phi}_0/\omega)\sin[\omega(t-t_0)]\\ & + (6\xi_0/R)\sin[\omega(t-t_0)] + \left(2a_\xi\big/\omega^2 R\right)\sin[\omega(t-t_0)]\\ & - \left(4a_\phi\big/\omega^2 R\right)\cos[\omega(t-t_0)]\end{aligned} \tag{4b}$$

where the initial state $(\xi, \phi, \dot{\xi}, \dot{\phi})$ of the solar sail at the initial time t_0 is defined by $(\xi_0, \phi_0, \dot{\xi}_0, \dot{\phi}_0)$. The solar sail velocity components can easily be obtained by differentiating Eqs. (4). It can be seen that, when starting from rest, the forcing terms will induce an azimuthal drift. There is quadratic term $3a_\phi/2R$ due to the azimuthal forcing term, as expected, but there is also a linear term $2a_\xi/\omega R$ due to the radial forcing term. This is a more subtle effect that occurs because, for a fixed sail pitch angle, inverse square solar gravity is modified by the radial, inverse square component of solar radiation pressure. The resulting modification to the effective radial force acting on the solar sail leads to a small increase in orbit period, which appears as an azimuthal drift.

III. Optimum Drift Trajectories

To generate an azimuthal repositioning maneuver, the mission requirements must be considered to set the boundary conditions of the problem. First, rest-to-rest trajectories will be considered so that at the maneuver final time t_f the solar sail is returned to a circular, one-year orbit. Therefore, the solar state $(\xi, \phi, \dot{\xi}, \dot{\phi})$ begins at initial time t_0 as $(0, 0, 0, 0)$, whereas at final time t_f the required solar sail state is defined as $(0, \Delta\phi, 0, 0)$, where $\Delta\phi$ is the change in azimuthal position of the solar sail relative to the Earth. A simple sail steering law will now be defined to induce this required change in azimuthal position $\Delta\phi$. To induce drift ahead of the Earth on a leading trajectory, the solar sail will be pitched at some fixed angle $-\alpha^+$ for a duration $(t_f - t_0)/2$ and then $+\alpha^+$ for a duration $(t_f - t_0)/2$. In doing so, the solar sail will reduce its heliocentric orbit radius, hence, shortening its orbit period and so drift ahead of the Earth, before returning to a circular one-year orbit. Similarly, to induce drift behind the Earth on a trailing trajectory, the solar sail will be pitched at some fixed angle $+\alpha^-$ for a duration $(t_f - t_0)/2$ and then $-\alpha^-$ for a duration $(t_f - t_0)/2$. In doing so, the solar sail will increase its heliocentric orbit radius, hence, lengthening its orbit period and so drift behind the Earth.

It can be shown from Eq. (2a) that using the control law just defined, the solar sail will only return to a circular orbit, and so $\dot{\xi}(t_f) = 0$, if $\omega(t_f - t_0) = 4m\pi$, for some positive integer m. Therefore, if $m = 1$ a rest-to-rest maneuver will require a minimum of two-years duration. When $\omega(t_f - t_0) = 4m\pi$ is substituted in Eq. (4), it can be shown that

$$\xi(t_f) = \dot{\xi}(t_f) = \dot{\phi}(t_f) = 0 \tag{5}$$

Similarly, if we now set $m = 1$ to define a two-year rest-to-rest maneuver, and use the sail steering law just described, the total change in azimuthal position of the solar sail relative to the Earth on the leading trajectory is found to be

$$\Delta\phi^+ = 4\pi\beta\cos^2\alpha^+(3\pi\sin\alpha^+ - 2\cos\alpha^+) \tag{6a}$$

whereas using the alternate sail steering law, the total change in azimuthal angle relative to the Earth on the trailing trajectory is found to be

$$\Delta\phi^- = -4\pi\beta\cos^2\alpha^-(3\pi\sin\alpha^- + 2\cos\alpha^-) \tag{6b}$$

The sail steering angles can now be optimized to maximize the change in azimuthal position relative to the Earth over the fixed two-year maneuver duration. Using Eqs. (3) and differentiating Eqs. (6) with respect to the sail pitch angle yields the optimum pitch angle for the advancing trajectory as

$$\cos^2\alpha^+ = [2/(4 + 9\pi^2)]\left(1 + 3\pi^2 - \sqrt{1 + 2\pi^2}\right) \tag{7a}$$

whereas the optimum pitch angle for the trailing trajectory is given by

$$\cos^2\alpha^- = [2/(4 + 9\pi^2)]\left(1 + 3\pi^2 + \sqrt{1 + 2\pi^2}\right) \tag{7b}$$

where $\alpha^+ \sim 41.48$ deg and $\alpha^- \sim 29.43$ deg. The functional form of Eqs. (6) is shown in Fig. 2 with the optimum sail steering angles indicated. It can be seen that an azimuthal drift will also be induced with $\alpha = 0$. Again, for a fixed-sail pitch angle, inverse square solar gravity is modified by the radial, inverse square component of solar radiation pressure. The resulting modification to the effective radial

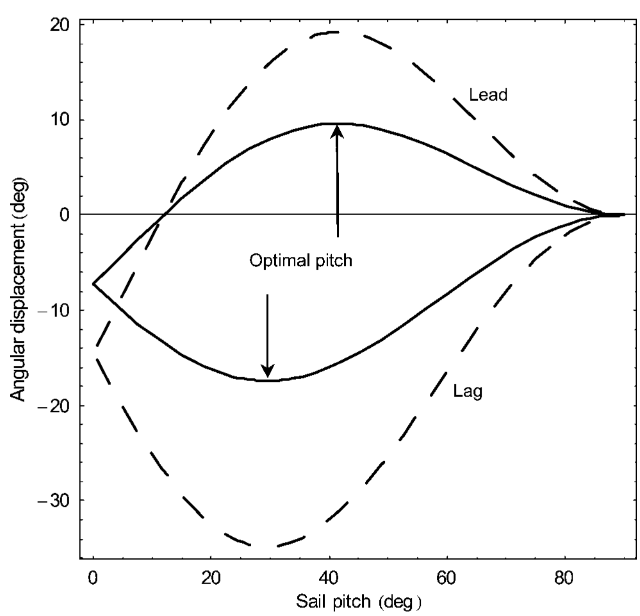

Fig. 2 Angular displacement of solar sail as a function of sail pitch for $m = 1$: ——, $\beta = 0.005$ and – – –, $\beta = 0.01$.

force acting on the solar sail leads to a small increase in orbit period, which appears as an azimuthal drift. Also note that the solar sail can be used to stationkeep at any azimuthal distance around the Earth's orbit because the solar sail can generate an artificial equilibrium point at the end of a lead or lag trajectory.[3]

IV. Comparison with Chemical Propulsion

The Δv requirements to perform azimuthal repositioning using impulsive chemical propulsion will now be investigated, so that $a_\xi = 0$ and $a_\phi = 0$. In addition, a rest-to-rest maneuver will again be considered so that the initial state $(\xi, \phi, \dot{\xi}, \dot{\phi})$ of the spacecraft is defined by $(0, 0, 0, \Delta v/R)$, where a transverse impulse Δv has induced an initial azimuthal drift rate of $\Delta v/R$ at time t_0. Equations (4) then reduce to

$$\xi(t) = (2\Delta v/\omega)\{1 - \cos[\omega(t - t_0)]\} \tag{8a}$$

$$\phi(t) = -(3\Delta v/R)(t - t_0) + (4\Delta v/\omega R)\sin[\omega(t - t_0)] \tag{8b}$$

To provide a direct comparison with the solar sail azimuthal repositioning maneuver we will again define $\omega(t_f - t_0) = 4\pi$ so that $\xi(t_f) = 0$ and

$$\Delta\phi = -12\pi\Delta v/\omega R \tag{9}$$

Again, if $\Delta v \succ 0$, then $\Delta\phi \prec 0$ because the orbit semimajor axis will be raised, leading to an increase in orbit period relative to the Earth and, hence, a trailing trajectory. For a rest-to-rest maneuver, two transverse impulses of equal magnitude and opposite sign are required, so that the total Δv required to maneuver the spacecraft is given by

$$\|\Delta v_T\| = (\omega R/6\pi)\Delta\phi \tag{10}$$

Therefore, assuming a specific impulse and propellant tank mass fraction for the propulsion system, the mass of the chemical propulsion subsystem can be obtained for a given final spacecraft mass. Similarly, Eqs. (6) can be used along with Eqs. (3) and (7) to determine the required solar sail lightness number for a given azimuthal displacement on either a lead or lag trajectory. For a given sail assembly loading (mass per unit area of the sail film and booms), the total sail mass can then by obtained.[1]

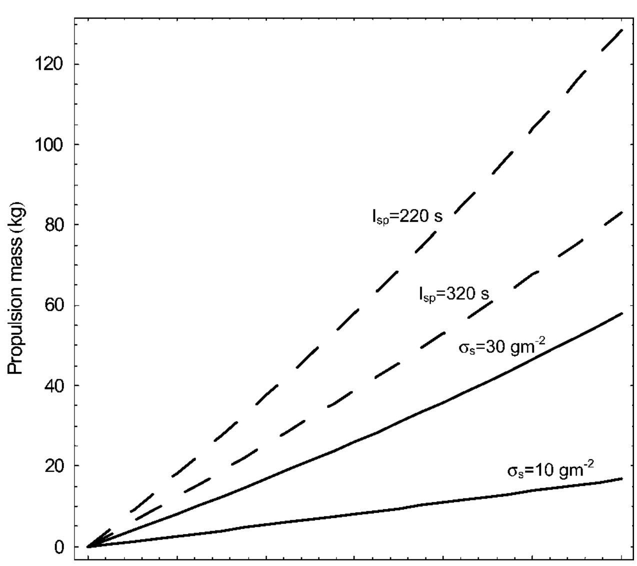

Fig. 3 Propulsion subsystem mass requirements for solar sail and chemical propulsion with a final payload mass of 250 kg: ——, solar sail propulsion and – – –, chemical propulsion.

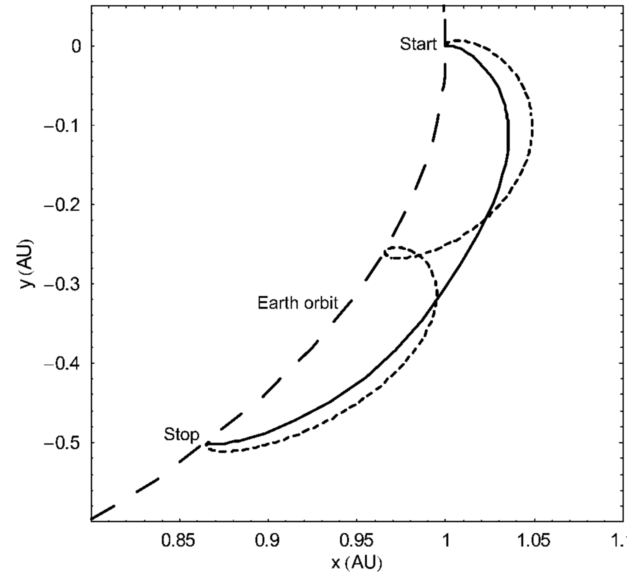

Fig. 4 Two-year azimuthal repositioning through 30 deg on a lag trajectory: ——, solar sail propulsion and – – –, chemical propulsion.

Figure 3 shows the propulsion system mass requirements for both chemical and solar sail propulsion for a range of specific impulses and sail assembly loadings. The final mass to be delivered is assumed to be 250 kg (similar to the STEREO mission) and a propellant tank mass fraction of 10% has been assumed. A sail assembly loading of 30 $g \cdot m^{-2}$ represents current ground tested capabilities, whereas 10 $g \cdot m^{-2}$ represent likely near-term performance capability. Similarly, a specific impulse of 220 s is representative of a monopropellant hydrazine system, whereas 330 s is representative of a bipropellant hydrazine/nitrogen–tetroxide system. It can be seen that the solar sail propulsion system mass is significantly lower than that for chemical propulsion, particular if a sail assembly loading of order 10 $g \cdot m^{-2}$ is available.

A two-year azimuthal repositioning maneuver through a lag of 30 deg is shown in Fig. 4 for both solar sail and chemical propulsion. Such a mean drift rate of order 15 deg/year on the lag trajectory is equivalent to the STEREO mission trajectory. Again for a delivered mass of 250 kg, a bipropellant chemical propulsion system would requires a mass of order 130 kg, whereas a solar sail with a assembly loading of 10 $g \cdot m^{-2}$ would require a mass of order 17 kg, corresponding to a 41×41 m solar sail. Further repositioning maneuvers would show even greater benefits for solar sail propulsion.

V. Conclusions

It has been shown that a small solar sail can be used to reposition a large science payload with a smaller propulsion system mass than chemical propulsion. The maneuver is well suited to solar sailing because there is a relatively long duration over which solar radiation pressure can act, resulting in only a modest sail size and mass. Such open-ended propulsion allows multiple-rest-to-rest maneuvers and controlled stationkeeping.

Acknowledgment

This work was undertaken with the kind support of the Leverhulme Trust.

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